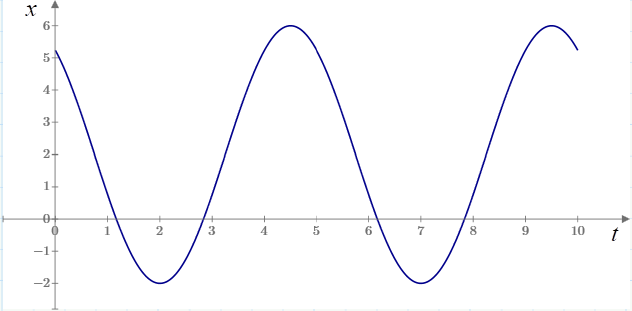
**Assignment 1 Solutions Due 1/24**

**Problem 1**. A 19th century amusement park ride consists of a spring attached to a box on wheels. You stretch the spring to some point, jump into the box, and then off you go! The following x(t) plot from that ride was recently found in a museum.



(a) What is the equilibrium length of the spring?

ℓ = 2, as it’s midway between the two extremes: 6 and -2.

(b) What is the period? Frequency? Angular frequency?

T = 5s, f = 1/T = 0.2Hz, ω = 2π/T = 2π/5 rad/s = 1.26 rad/s

(c) What is the amplitude?

A = half the distance between the two extremes = 4.

(d) What is the phase constant (approximately)?

Phase constant is hard to work out from the description given in notes. But we can ascertain it by using the initial conditions.



Another way is to observe that the plot hits its first nadir at t = 2s, which implies at t = 2s, cos(ωt+φ0) must be -1 → ω∙2 + φ0 = π → φ0 = π - 2ω = π – (2π/5)(2) = π - 4π/5 = π/5 = 0.63.

(e) Write down an expression for x(t), if you don’t mind.

Not at all. 

(f) What is the maximum speed?

Simply this:



(g) What is the maximum acceleration?

That’d be…

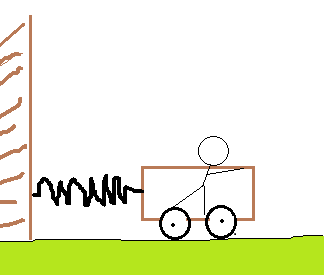


(h) If the surface of the box were smooth, you’d be prone to slip back and forth. Indicate on the graph above, at what times you be most prone to do so. What minimum coefficient of static friction would there have to be between you and the box to prevent you from slipping?

Slipping would tend to happen at the points where a is largest – i.e. at the peaks and nadirs. Coefficient of friction would have to satisfy:



**Problem 2**. Consider that amusement park spring ride again. Say the spring constant is k = 300N/m, and equilibrium length of the spring is ℓ = 3m. Someone holds the box at x = 2m, and you take a running start and jump into the box with velocity v = 7m/s. Your mass is m = 60kg. And the box’s mass is m = 40kg.



(a) Assuming the collision between you and the box happens instantaneously (and you stick together), what is yall’s (you and box) initial velocity? Have to use conservation of something to work this out!

Gonna use conservation of momentum,



(b) What will be your angular frequency? Period?

Well, partner,



(c) What is the amplitude?

We’ll use initial conditions. Note x(t) = ℓ + Acos(ωt+φ0) = 3 + Acos(1.73t + φ0). And v(t) = dx/dt = -1.73Asin(1.73t + φ0). So,



So,



(d) What is the phase constant?

Pretty easy, I’d say,



(e) Write down an expression for x(t), por favor.

Por supuesto. 

(f) What is your maximum and minimum positions?

Xmax = 3+2.63 = 5.63m

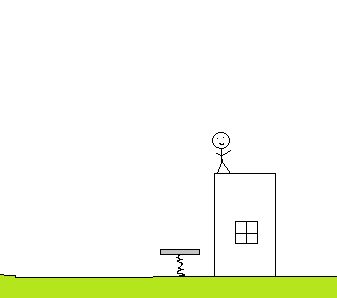
Xmin = 3-2.63 = 0.37m

(g) What is the maximum speed?

Simply this:



**Problem 3**. Suppose you (mass m = 70kg) step off an 5m tall platform, onto 2m tall spring (k = 2000N/m). Starting time from when you hit the spring…



(a) what will be new equilibrium position about which you oscillate?

Equilibrium is where forces balance. So



(b) what will be your period?

Period is



(c) what will be your amplitude of oscillation?

Could do this with energy conservation, but we’ll do it as in class. First we need your velocity when you hit the spring. This is, via energy conservation:



We must account this as negative, since ‘+’ direction is that of the stretch of the spring, which would be up. So your initial position and initial velocity are x0 = 2m, and v0 = -7.67m/s. Now fill these into the spring equation x(t) = xeq. + Acos(ωt + φ0) = 1.66 + Acos(5.35t + φ0):



We can now solve for A:



(d) what will be your phase constant?

And for the phase constant,



(e) write down an expression for y(t), vy(t), and ay(t).

So,



(f) What will be your position, velocity, acceleration at time t = 1s?

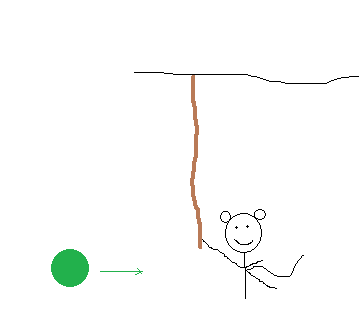
Well, we plug t = 1 into these equations:



(g) would only gravity suffice to keep you on the spring platform?

No, as gravity can only give you an acceleration of g = -9.8m/s2, whereas the maximum acceleration the spring would attempt to impart is greater than this.

**Problem 4.** A monkey (m = 10kg) hangs on a vine (can treat as a uniform board with mass m = 6kg) of length ℓ = 5m. You throw a watermellon (m = 3kg) horizontally at speed v = 18m/s, which the monkey catches. (a) What will be the monkey’s period of oscillation? (b) What would be the period if the speed of the watermelon were doubled?



(a) So the formula for the period is:



Now



And the center of mass would be:

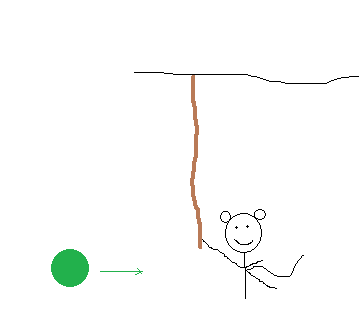


So,



(b) Same.

**Problem 5.** A monkey (m = 10kg) hangs on a massless vine of length ℓ = 5m. You throw a watermellon (m = 3kg) horizontally at speed v = 18m/s, which the monkey catches. Supposing the catch happens instantaneously….



(a) What is monkey’s initial angle?

Uh, 0°. Duh.

(b) What will be the monkey’s initial angular after catching the watermelon? \*Hint: gotta use conservation of something here!

Mmwa ha ha. So conservation of angular momentum would say:



(c) What will be the monkey’s period of oscillation?

Period of oscillation comes from ω,



(d) What maximum angle will the monkey reach?

Could use energy, but again, we’ll do as in class…so θ(t) = Acos(ωt+φ0) = Acos(1.4t+φ0). And initial conditions are θ0 = 0, ω0 = 0.83. So,



Doing the usual,



(e) What will be the monkey’s phase constant?

and,

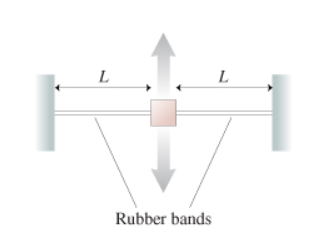


(f) Write down expressions for the monkey’s angle, angular velocity, and angular acceleration as a function of time.

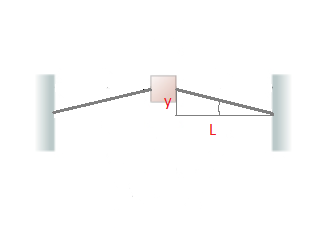
Therefore,



**Problem 6.** Consider a mass, m, attached to two rubber bands each under tension T. If you were to displace the mass upwards a small amount, it would oscillate back and forth with some frequency f.



(a) write down an expression for the net force on the mass, ignoring gravity, if it is displaced a distance y above the horizontal, assuming that the tension in the rubber bands remains constant. Your expression should only contain L, T, and y. Also, use the small angle approximation sinθ ≈ tanθ.



We have:



(b) Wriite down N2L for the mass.



(c) Plug in the trial solution y(t) = Acos(ωt+φ0) and solve for ω, and then the frequency, symbolically.

So,



And so the frequency is:

